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RESEARCH MEMORANDUM

ON SLENDER-BODY THEORY AT TRANSONIC SPEEDS

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NATIONAL ADVISORY COMMITTEE
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SUMMARY

The basic ideas of the slender-body approximation have been applied to the nonlinear transonic-flow equation for the velocity potential in order to obtain some of the essential features of slender-body theory at transonic speeds. The results of the investigation are presented from a unified point of view which demonstrates the similarity of slender-body solutions in the various Mach number ranges. The primary difference between the results in the different flow regimes is represented by a certain function which is dependent upon the body area distribution and the stream Mach number. The transonic area rule and some conditions concerning its validity follow from the analysis.

INTRODUCTION

Slender-body theory originated with Munk's paper (ref. 1) in 1924 in which the forces on slender airships were calculated for low-speed flight. In 1938 Tsien (ref. 2) pointed out that Munk's airship theory also applied to the flow past inclined, pointed bodies at supersonic speeds. The subject gained new importance in 1946 with the appearance of Jones's paper (ref. 3) in which it was shown that the basic ideas of the slender-body approximation could be used to calculate the forces on slender lifting wings at both subsonic and supersonic speeds provided that proper account was taken of trailing-vortex sheets. Since Jones's paper, the subject has received wide treatment in the literature. In an important paper in 1949, Ward (ref. 4) developed a general unifying theory for the flow past smooth, slender, pointed bodies at supersonic speeds which contains as special cases the lifting planar wings of Jones and the slender nonlifting bodies treated by Von Kármán (ref. 5). The corresponding problem at subsonic speeds has been examined by Adams and Sears (ref. 6) who also extended the slender-body concepts to shapes which are "not so slender." Lighthill (ref. 7) has given a method for calculating the flow past bodies with discontinuities in slope. Keune (ref. 8) has developed solutions for slender wings with thickness and various lifting configurations have been treated by Heaslet, Spreiter, Lomax, Ribner, and others (refs. 9 to 14).

The slender-body theory presented in references 2 to 14 has been based upon the linearized equation for the velocity potential. In the present paper, the basic ideas of the slender-body approximation are applied to the nonlinear transonic equation for the velocity potential in order to obtain some of the essential features of slender-body theory at transonic speeds. The attempt has been made to present the results from a unified point of view which demonstrates the similarity of the slender-body solutions in the various Mach number ranges.

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SLENDER-BODY APPROXIMATION

Slender-body theory deals with that class of shapes whose length is large compared with any lateral dimension. For such shapes at both subsonic and supersonic speeds, the flow in planes normal to the stream direction can be approximated by solutions of Laplace's equation. The justification is that for very slender wings or bodies the variation of the geometrical properties in the stream direction is small and, consequently, the rate of change of the longitudinal component of the velocity in the stream direction is also small. The various slender-body solutions have all been developed on the basis of the linearized potential equation. However, a similar development can be made on the basis of the nonlinear transonic equation.

The simplest differential equation for the disturbance potential Φ which is generally valid at transonic speeds (ref. 15, for example) is

$$\left[1 - M^2 - (\gamma + 1)M^2\Phi_x\right]\Phi_{xx} + \Phi_{rr} + \frac{\Phi_r}{r} + \frac{\Phi_{\theta\theta}}{r^2} = 0 \tag{1}$$

where x, r, and ϑ are cylindrical coordinates, M is the stream Mach number, and γ is the ratio of specific heats at constant pressure and constant volume. With the introduction of the dimensionless coordinates ξ and η by $x = l\xi$ and $r = b\eta$, and of the dimensionless potential ϕ by $\Phi = \frac{b^2}{l}\phi(\xi,\eta,\vartheta)$, the transonic potential-flow equation becomes

$$\left(\frac{b}{l}\right)^2\left[1 - M^2 - (\gamma + 1)M^2\left(\frac{b}{l}\right)^2\phi_\xi\right]\phi_{\xi\xi} + \phi_{\zeta\zeta} + \frac{\phi_\zeta}{\zeta} + \frac{\phi_{\theta\theta}}{\zeta^2} = 0 \tag{2}$$

where l is a characteristic length and b is a characteristic width such as the largest lateral dimension of the configuration. For sufficiently small values of the width parameter b/l, it appears that the terms involving derivatives in the stream direction can be neglected to obtain the result that the flow satisfies Laplace's equation

$$\Phi_{rr} + \frac{1}{r}\Phi_r + \frac{1}{r^2}\Phi_{\vartheta\vartheta} = 0 \tag{3}$$

in the cross-flow plane. Equation (3) represents the slender-body approximation to equation (1). Some conditions will be determined subsequently which are necessary in order for solutions of equation (3) to be approximate solutions of equation (1).

The boundary conditions for the flow about a body in a uniform stream are the vanishing of the disturbance velocities at infinity and

$$\frac{\partial\Phi}{\partial n} = \frac{dn}{dx}(1 + \Phi_x) \approx \frac{dn}{dx}$$

on the body where n is in the direction normal to the body contour in the cross-flow plane. For flows satisfying Laplace's equation in the cross-flow plane, the surface boundary condition can be integrated (ref. 4, for example) to give

$$\int_C \frac{\partial\Phi}{\partial n}\, dt = S'(x) \tag{4}$$

where dt is an element length in the direction of the tangent to any contour C in the cross-flow plane enclosing the body, $S(x)$ is the cross-sectional area distribution of the body, and the prime denotes differentiation with respect to the indicated argument.

The slender-body solution of equation (1) can be represented by a solution of equation (3) plus a function of integration. Since equation (3) is independent of Mach number, the form of the solution is identical with the known slender-body solutions for subsonic and supersonic flow. The slender-body analyses of references 4 and 6 have established that the solution can be represented by a distribution of sources and higher order singularities on the axis; an equivalent form is given by a distribution of sources in the region of the cross-flow-plane interior to the surface boundary. The slender-body solution is then expressed as

$$\Phi = \frac{b^2}{l}\left[\frac{1}{2\pi}\iint_s f(\rho_1,\vartheta_1;\xi)\log\sqrt{(\rho\cos\vartheta - \rho_1\cos\vartheta_1)^2 + (\rho\sin\vartheta - \rho_1\sin\vartheta_1)^2}\,\rho_1\,d\rho_1\,d\vartheta_1 + g(\xi)\right]$$

where $x = l\xi$, $r = l\rho$, $g(\xi)$ is an arbitrary function of integration, and the region of integration s is the nondimensional cross-sectional area defined by $S(x) = b^2 s(\xi)$. By adding and subtracting $\log\rho$ from the integrand, the potential for the source distribution is expressed as

$$\Phi = \frac{b^2}{l}\left[\frac{1}{2\pi}\log\rho\iint_s f(\rho_1,\vartheta_1;\xi)\rho_1\,d\rho_1\,d\vartheta_1 + \right.$$

$$\left.\frac{1}{2\pi}\iint_s f(\rho_1,\vartheta_1;\xi)\log\sqrt{\left(\cos\vartheta - \frac{\rho_1}{\rho}\cos\vartheta_1\right)^2 + \left(\sin\vartheta - \frac{\rho_1}{\rho}\sin\vartheta_1\right)^2}\,\rho_1\,d\rho_1\,d\vartheta_1 + g(\xi)\right] \tag{5}$$

The last two terms of equation (5) make no contribution to the contour integral in the integrated form of the surface boundary condition (eq. 4) and the double integral in the first term is determined as $s'(\xi)$. The potential is then expressed as

$$\Phi = \frac{b^2}{l}\left[\frac{s'(\xi)}{2\pi}\log\rho + \frac{1}{2\pi}\iint_s f(\rho_1,\vartheta_1;\xi)\log\sqrt{\left(\cos\vartheta - \frac{\rho_1}{\rho}\cos\vartheta_1\right)^2 + \left(\sin\vartheta - \frac{\rho_1}{\rho}\sin\vartheta_1\right)^2}\,\rho_1\,d\rho_1\,d\vartheta_1 + \right.$$

$$\left. g(\xi)\right]$$

$$= \frac{b^2}{l}\left[\varphi(\rho,\vartheta;\xi) + g(\xi)\right] \tag{6}$$

where φ denotes the solution of Laplace's equation in the cross-flow plane with ξ appearing as a parameter introduced by the geometry of the cross section at ξ. The function φ, being independent of the stream Mach number, can be evaluated for incompressible flow past the shape under consideration.

A necessary condition for equation (6) to be an approximate solution of equation (1) is that the terms neglected in equation (1) be small compared with those retained. By assuming for the moment that $\log \rho$ is the only singular term in Φ, the ratio of the term $\left[1 - M^2 - (\gamma + 1)M^2\Phi_x\right]\Phi_{xx}$, which is neglected in the slender-body approximation, to any of the remaining terms in equation (1) is of the order

$$\left(\frac{r}{l}\right)^2 \left\{(1 - M^2)\left[O\left(\log \frac{r}{l}\right) + O(1)\right] + \left(\frac{b}{l}\right)^2\left[O\left(\log^2 \frac{r}{l}\right) + O\left(\log \frac{r}{l}\right) + O(1)\right]\right\}$$

where $O(\ )$ denotes order of and the nonsingular terms are denoted by $O(1)$. This ratio can be made smaller than any prescribed value ϵ by restricting the solution to the interior of a cylinder of some radius, say R/l. For given values of M and ϵ, the radius of this cylinder increases with decreasing b/l and the ratio R/b approaches infinity as b/l approaches zero. Moreover, for given values of ϵ and R/l, larger values of b/l are permitted as M approaches 1.

From equation (6) it is apparent that $S'(x/l)$ and its derivatives must be finite in order to satisfy the requirement of small disturbances. Moreover, an additional restriction on the asymmetry of the body is sometimes required (ref. 4), particularly for lifting configurations - namely, the radius of curvature of the body boundary in the cross-flow plane must be $O(b)$ where the boundary is convex outward. The restrictions on body shape imposed by the function $g(x/l)$ will be considered subsequently after the nature of this function is established.

The function $g(x/l)$ is determined from considerations involving the complete transonic differential equation (eq. 1) and, consequently, is dependent upon the stream Mach number. Examination of equation (6) at $\frac{r}{l} = \frac{R_o}{l} < \frac{R}{l}$ shows that the contribution of the asymmetric term of the source distribution to the potential can be made smaller than any prescribed value by making b/R_o sufficiently small $\left(\frac{\rho_1}{\rho_o} = \frac{r_1}{R_o} \leqq \frac{b}{R_o}\right)$. To this order of approximation, then, the flow field external to R_o is axisymmetric. Moreover, there is an axisymmetric flow which approximately matches the pressure and flow direction of the slender-body

solution at $r = R_o$. The potential Φ_o for this associated axisymmetric flow satisfies equation (1) and is expressed symbolically as

$$\Phi_o = \frac{b^2}{l}\left[\frac{s_o'(x/l)}{2\pi}\log\frac{r}{l} + g_o(x/l, r/l)\right] \tag{7}$$

where the characteristic width b and length l are taken the same as for the asymmetric shape. In equation (7), $s_o(x/l)$ is the area distribution of the associated body of revolution which has the same outer flow $(r \geqq R_o)$ as the asymmetric shape. If this axisymmetric flow satisfies the slender-body conditions, then g_o is independent of r for $r < R$. In order for the radial derivatives to match at $r = R_o$, $s_o(x/l)$ must be equal to $s(x/l)$; that is, the associated body of revolution must have the same axial distribution of cross-sectional area as the asymmetric shape. In order for the pressure distributions to match at $r = R_o$, $g_o(x/l)$ must be the same as $g(x/l)$. Thus, $g(x/l)$ is the same function as that for a body of revolution having the same axial distribution of cross-sectional area.

In the preceding discussion the region of validity of the slender-body approximation to Φ_o was tacitly assumed to be at least as large as that for Φ. This condition is certainly true since the singular terms in the two solutions are the same.

In the slender-body approximation the term $\left[1 - M^2 - (\gamma + 1)\Phi_x\right]\Phi_{xx}$ is required to be small compared with any of the other terms in the transonic differential equation. If this condition is to be satisfied in the neighborhood of weak shock waves where $g'(x/l)$ would be required to have a jump proportional to the pressure rise and $g''(x/l)$ would be infinite, the quantities (which were included in the terms denoted as $O(1)$)

$$g''(x/l)\left[1 - M^2 - (\gamma + 1)M^2\Phi_{o_x}\right] \tag{8}$$

and

$$g''(x/l)\frac{\partial}{\partial x}\iint_S f(\rho_1,\vartheta_1;x/l)\log\sqrt{\left(\cos\vartheta - \frac{\rho_1}{\rho}\cos\vartheta_1\right)^2 + \left(\sin\vartheta - \frac{\rho_1}{\rho}\sin\vartheta_1\right)^2}\,\rho_1\,d\rho_1\,d\vartheta_1 \tag{9}$$

must be bounded there. Thus, at shock waves the coefficients of $g''(x/l)$ in expressions (8) and (9) must vanish. In the first of these expressions, which is axisymmetric, the coefficient of $g''(x/l)$ vanishes for a local Mach number of 1. Moreover, the average value of the local Mach number ahead and behind a weak normal shock wave is 1. With this interpretation of shock waves, then, the coefficient of $g''(x/l)$ vanishes and the solution should represent the flow in the neighborhood of weak normal shocks. Also, since $g(x/l)$ is the same function for both a slender configuration and its associated body of revolution, their shock-wave systems would have the same strength and location. In order for expression (9) to be bounded in the neighborhood of a shock wave, the double integral must vanish. Since this double integral gives rise to asymmetric shapes and is zero for chordwise locations where the body is axisymmetric, the additional restriction is obtained that the body cross section must be circular in the vicinity of shock waves.

The slender-body solutions in the various Mach number ranges are similar in that they are all represented by equation (6) although the function $g(x/l)$ differs for the various speed ranges. Analytic expressions for $g(x/l)$ have been obtained for supersonic and subsonic flows by considering general solutions of the complete linearized equation satisfying the boundary condition of vanishing disturbance velocities at infinity. These solutions were then expanded in the neighborhood of the body to evaluate $g(x/l)$. Ward (ref. 4) has determined this function for supersonic flows as

$$g(x/l) = \frac{1}{2\pi}\left[\frac{s'(x/l)}{2}\log\left(\frac{M^2 - 1}{4}\right) - \int_0^{x/l} s''(\xi_1)\log\left(\frac{x}{l} - \xi_1\right)d\xi_1\right]$$

The corresponding result at subsonic speeds was obtained by Adams and Sears (ref. 6) as

$$g(x/l) = \frac{1}{2\pi}\left[\frac{s'(x/l)}{2}\log\left(\frac{1 - M^2}{4}\right) - \frac{1}{2}\int_0^{x/l} s''(\xi_1)\log\left(\frac{x}{l} - \xi_1\right)d\xi_1 + \frac{1}{2}\int_{x/l}^{1} s''(\xi_1)\log\left(\xi_1 - \frac{x}{l}\right)d\xi_1\right]$$

where the body extends from $x = 0$ to $x = l$. Although an analytic expression for $g(x/l)$ at transonic speeds is not known, it has been established that the stream Mach number enters the solution only through $g(x/l)$ and that the only geometric property of the body influencing

this function is the area distribution. The transonic similarity rule for bodies of revolution (ref. 15 or 16) shows that $g(x/l)$ can be expressed in the form

$$g(x/l) = \frac{s'(x/l)}{4\pi} \log\left[(\gamma + 1)M^2\left(\frac{b}{l}\right)^2\right] + f(x/l;K)$$

where the similarity parameter is

$$K = \frac{1 - M^2}{(\gamma + 1)M^2\left(\frac{b}{l}\right)^2}$$

AERODYNAMIC FORCES

Since the slender-body solutions are all represented by equation (6), formal expressions for the aerodynamic forces can be determined which are valid throughout the Mach number range. Consequently, many of the essential features of slender-body theory at transonic speeds can be obtained without resorting to detailed calculations.

Lift

The most significant difference between the slender-body solutions at subsonic, transonic, and supersonic speeds is that the function $g(x/l)$ differs in these various speed ranges. However, the term in the pressure arising from the function $g(x/l)$ makes only a uniform contribution to the pressure at any value of x and, therefore, cannot affect the lift distribution or the lift. Thus, within the slender-body approximation, the lift distribution depends only upon the function φ and, consequently, is independent of the stream Mach number. Several investigators (Robinson and Young (ref. 17) and Heaslet, Lomax, and Spreiter (ref. 9), for example) have previously noted that the linearized slender-body theory gave consistent results, even at a Mach number of 1, for planar systems.

According to slender-body theory, the lift distribution can be obtained completely from solutions of Laplace's equation in the cross-flow plane. Since this equation is linear, the lift is proportional to the angle of attack even at transonic speeds. Ward has obtained an especially simple form for the drag due to lift in which

$$D_L = \frac{1}{2} \alpha L$$

where α is the angle of attack measured from zero lift and L is the lift.

Drag

By computing the momentum change of the fluid passing through a cylinder enclosing the body, the drag is determined as

$$\frac{D}{q} = \frac{D_b}{q} - \left(\frac{b^2}{l}\right)^2 \left[2 \int_0^1 s'(\xi) g'(\xi) d\xi + \int_{C'} \varphi \frac{\partial \varphi}{\partial n} dt\right] \tag{10}$$

where the body extends from $\xi = 0$ to $\xi = 1$, C' denotes the contour of the body at the stern which in the case of wings or wing-body combinations includes the trailing-vortex wake, q is the stream dynamic pressure, and D_b is the base drag. Equation (10) is valid throughout the Mach number range provided the appropriate forms of the function $g(x/l)$ are employed. The line integral is zero for nonlifting configurations if the body is closed or if the body ends in a cylindrical section whose elements are parallel to the stream. The effect of Mach number (excluding the variation of base drag with Mach number) is contained in the term involving $g(x/l)$.

When the subsonic form of $g(x/l)$ is used in equation (10), the correct result is obtained that the drag of nonlifting configurations is zero. By using the supersonic form of $g(x/l)$, the drag varies with Mach number like $\left[s'(1)\right]^2 \log(M^2 - 1)$. For pointed bodies, or for bodies which end in a cylindrical section, the supersonic slender-body theory indicates that the drag is independent of Mach number. For bodies which do not satisfy these conditions, the supersonic result indicates that the drag approaches infinity as the Mach number approaches 1. These results from linear theory cannot be considered satisfactory at transonic speeds since they give a discontinuity in the drag as the Mach number is increased through 1; whereas experimental data show that the drag starts to increase rapidly at a subsonic Mach number and varies smoothly through 1. However, the few known solutions of the nonlinear transonic-flow equation are in good agreement with experiment in this regard. Consequently, the drag rise of slender shapes should be correctly approximated by equation (10) when the transonic form of $g(x/l)$ is employed in the drag equation.

Transonic Area Rule

The body shape enters into the function $g(x/l)$ only as a function of the cross-sectional area distribution throughout the Mach number range. This property of the slender-body solutions leads to an important result even though the analytic expression for $g(x/l)$ is not known at transonic speeds. Examination of equation (10) shows that the body cross-sectional shape enters into the slender-body drag expression only through the contour integral evaluated at the stern of the configuration. For a fixed geometry at the base, then, the drag of nonlifting configurations depends only on the axial distribution of the body cross-sectional area and is independent of the cross-sectional shape. Thus, within the slender-body approximation, the drag of a nonlifting configuration is the same as that of the associated body of revolution having the same streamwise distribution of cross-sectional area provided the base geometry is fixed. It is in this sense that an equivalent body of revolution is associated with a wing-body combination. This result, often referred to as the area rule, is especially significant at transonic speeds where larger values of the width parameter b/l are permitted than in other speed ranges.

The property of the dependence of the drag upon the distribution of cross-sectional area has previously been obtained by Ward (ref. 4) and Graham (ref. 18) for supersonic flow and has been observed experimentally by Whitcomb (ref. 19, for example) at transonic speeds. The importance of this result was first noted by Whitcomb who demonstrated that the area rule could be used as a basis for the design of low-drag wing-body combinations at transonic speeds.

From the preceding development, the transonic area rule is subject to the restrictions of slender-body theory with the additional condition that the base geometry be fixed. These restrictions imply that modifications to the equivalent body of revolution must be performed in the region between shock waves. However, the seriousness of violating this condition is not well understood at the present time. For example, schlieren photographs seem to indicate that, even in cases where the presence of the wing affects the strength of the shock, the average strength of the shock may be close to that for the equivalent body of revolution.

Langley Aeronautical Laboratory,
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